Geometric Mechanics

Homework 11

Due on December 9

1. **Twin paradox**: Twins Alice and Bob part on their 20^{th} anniversary: while Alice stays on the Earth (which is approximately an inertial frame), Bob leaves at 80% of the speed of light towards Planet X, 8 light-years away from the Earth, which he therefore reaches 10 years later (as measured in the Earth's frame). After a short stay, Bob returns to the Earth, again at 80% of the speed of light. Consequently, Alice is 40 years old when they meet again.

   (a) How old is Bob at this meeting?

   (b) How do you explain the asymmetry in the twin's ages? Notice that, from Bob's point of view, he is the one who is stationary, while the the Earth moves away and back again!

   (c) Imagine that each twin has a very powerful telescope. What does each of them **see**? In particular, how much time elapses for each of them as they see their twin experiencing one year?

2. **Generalized twin paradox**: Let $p, q \in \mathbb{R}^4$ be two events connected by a timelike straight line segment l. Show that the proper time between p and q measured along l is bigger than the proper time between p and q measured along any other timelike curve connecting these two events. In other words, if an inertial observer and a (necessarily) accelerated observer separate at a given event and are rejoined at a later event, then the inertial observer always measures a bigger (proper) time interval between the two events.