# Geometric Mechanics

## Homework 7

*Due on November 10*

1. Let $(x^1, \ldots, x^n, p_1, \ldots, p_n)$ be the usual local coordinates on $T^*M$. Compute $X_{x^i}$, $X_{p_i}$, $\{x^i, x^j\}$, $\{p_i, p_j\}$ and $\{p_i, x^j\}$.

2. Show that in the Poincaré recurrence theorem the set of points $\alpha \in U$ such that $\psi_t(\alpha) \in U$ for some $t \geq T$ is dense in $U$.

3. Let $(M, \langle \cdot, \cdot \rangle)$ be a compact Riemannian manifold. Show that for each normal ball $B \subset M$ and each $T > 0$ there exist geodesics $c : \mathbb{R} \to M$ with $\|\dot{c}(t)\| = 1$ such that $c(0) \in B$ and $c(t) \in B$ for some $t \geq T$.