

# Geometric Mechanics

## Homework 6

*Due on November 3*

1. Let  $(M, \langle \cdot, \cdot \rangle)$  be a Riemannian manifold and  $p, q \in M$ . A curve of minimal length connecting  $p$  to  $q$ , with nonvanishing derivative, must be a critical point of the action determined by the Lagrangian  $L : TM \setminus Z \rightarrow \mathbb{R}$  given by

$$L(v) = \langle v, v \rangle^{\frac{1}{2}}$$

( $Z \subset TM$  is the zero section).

- (a) Show that such a curve is a reparameterized geodesic.
- (b) Compute the Hamiltonian function  $H : TM \setminus Z \rightarrow \mathbb{R}$ . Is  $L$  hyper-regular?

2. Consider the action of  $SO(3)$  on itself by left multiplication.

- (a) Show that the infinitesimal action of  $B \in \mathfrak{so}(3)$  is the **right**-invariant vector field determined by  $B$ , that is,

$$(X^B)_S = BS.$$

- (b) Use the Noether Theorem to show that the angular momentum  $p = SI\Omega$  of the free rigid body is constant.