

# Geometric Mechanics

## Homework 6

*Due on October 29*

1. Let  $(M, \langle \cdot, \cdot \rangle)$  be a Riemannian manifold. Show that the critical points of the arclength, i.e., of the action determined by the Lagrangian  $L : TM \rightarrow \mathbb{R}$  given by

$$L(v) = \langle v, v \rangle^{\frac{1}{2}}$$

(where we must restrict the action to curves with nonvanishing velocity), are reparameterized geodesics. What is the Hamiltonian function in this case?

2. Consider the action of  $SO(3)$  on itself by left multiplication.
  - (a) Show that the infinitesimal action of  $B \in \mathfrak{so}(3)$  is the **right**-invariant vector field determined by  $B$ , that is,
$$(X^B)_S = BS.$$
  - (b) Use the Noether Theorem to show that the angular momentum  $p = SI\Omega$  of the free rigid body is constant.