Geometric Mechanics

Homework 6

Due on November 2

1. Let $(M, \langle \cdot, \cdot \rangle)$ be a Riemannian manifold. Show that the critical points of the arclength, i.e., of the action determined by the Lagrangian $L : TM \to \mathbb{R}$ given by

$$L(v) = \langle v, v \rangle^{\frac{1}{2}}$$

(where we must restrict the action to curves with nonvanishing velocity), are reparameterized geodesics. What is the Hamiltonian function in this case?

2. Consider the action of $SO(3)$ on itself by left multiplication.

   (a) Show that the infinitesimal action of $B \in \mathfrak{so}(3)$ is the **right**-invariant vector field determined by B, that is,

$$\left(X^B\right)_S = BS.$$

   (b) Use the Noether Theorem to show that the angular momentum $p = SI\Omega$ of the free rigid body is constant.