Geometric Mechanics

Homework 13

Due on December 21

1. **A model of collapse:** Show that the radius of a spherical shell $r = r_0$ in a FLRW model changes with proper time in exactly the same fashion as the radius of a radially free-falling spherical shell in a Schwarzschild spacetime of mass parameter m moving with energy parameter E, provided that
$$\begin{cases} m = \alpha {r_0}^3 \\ E^2 - 1 = -k{r_0}^2 \end{cases}$$
Therefore these two spacetimes can be matched along the 3-dimensional hypersurface determined by the spherical shell's motion to yield a model of collapsing matter. Can you give a physical interpretation of this model?

2. Show that if we allow for a **cosmological constant** $\Lambda \in \mathbb{R}$, i.e. for an Einstein equation of the form
$$Ric = 4\pi\rho(2\nu \otimes \nu + g) + \Lambda g$$
then the equations for the FLRW models become
$$\begin{cases} \dfrac{\dot{a}^2}{2} - \dfrac{\alpha}{a} - \dfrac{\Lambda}{6}a^2 = -\dfrac{k}{2} \\[2ex] \dfrac{4\pi}{3}a^3\rho = \alpha \end{cases}$$
Analyze the possible behaviors of the function $a(t)$. (**Remark:** It is currently thought that there exists indeed a positive cosmological constant, also known as **dark energy**. The model favored by experimental observations seems to be $k = 0$, $\Lambda > 0$).