Algebraic and Geometric Methods in Engineering and Physics

Homework 9

Due on November 26

1. Consider the unit sphere,

$$S^2 = \{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 1\} \subset \mathbb{R}^3,$$

   as a topological space with the subspace topology. The **real projective plane** is the quotient space $\mathbb{R}P^2 = S^2/\sim$, where $\sim$ is the equivalence relation determined on S^2 by $(x, y, z) \sim (-x, -y, -z)$, equipped with the quotient topology.

   (a) Prove that $\mathbb{R}P^2$ is compact.

   (b) Show that the map $f : \mathbb{R}P^2 \to \mathbb{R}$ given by $f([(x, y, z)]) = x^2$ is is well defined.

   (c) Prove that the map $g : S^2 \to \mathbb{R}$ given by $g(x, y, z) = x^2$ is continuous.

   (d) Show that $g = f \circ \pi$, where $\pi : S^2 \to \mathbb{R}P^2$ is the canonical projection.

   (e) Prove that f is continuous.

2. Consider the interval $M = (-1, 1)$ as a metric space with the usual distance function $d(x, y) = |x - y|$. Show that M is closed and bounded but it is not compact.