

Algebraic and Geometric Methods in Engineering and Physics

Homework 7

Due on November 12

1. Show that the following choices of unitary matrices determine representations of the group $D_4 \equiv \{e, r, r^2, r^3, s, sr, sr^2, sr^3\}$, and compute their characters:

(a) $\varphi_r^{(1)} = \varphi_s^{(1)} = 1;$

(b) $\varphi_r^{(2)} = 1, \varphi_s^{(2)} = -1;$

(c) $\varphi_r^{(3)} = -1, \varphi_s^{(3)} = 1;$

(d) $\varphi_r^{(4)} = \varphi_s^{(4)} = -1;$

(e) $\varphi_r^{(5)} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \varphi_s^{(5)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$

(f) $\psi_r = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \psi_s = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$

2. Using the characters computed in the previous question, show that:

(a) $\varphi^{(1)}, \varphi^{(2)}, \varphi^{(3)}, \varphi^{(4)}$ and $\varphi^{(5)}$ are irreducible;

(b) $\psi \sim \varphi^{(1)} \oplus \varphi^{(3)} \oplus \varphi^{(5)}.$