

Algebraic and Geometric Methods in Engineering and Physics

Homework 5

Due on October 15

1. Let G be the symmetry group of a regular tetrahedron (that is, the group of isometries of Euclidean space that leave the tetrahedron invariant), and $H \subset G$ the rotation group of the tetrahedron (that is, the group of rotations of Euclidean space that leave the tetrahedron invariant).
 - (a) Show that G is isomorphic to S_4 .
 - (b) Show that H is isomorphic to A_4 .
 - (c) How many different colorings of a tetrahedron are there, if each face can be painted with any one of n possible colors?

(Hint: Consider the action of G on the vertices of the tetrahedron, and recall that any isometry of the Euclidean space is the composition of a finite number of reflections).

2. Consider a square window divided into $3 \times 3 = 9$ equally sized square glasses, as shown in the figure below. Each square glass can have one of n possible colors.
 - (a) How many different windows are there, up to reorientation (i.e. rotations and/or reflections)?
 - (b) Check explicitly that your answer gives an integer for each choice of n .

