Algebraic and Geometric Methods in Engineering and Physics

Homework 4

Due on October 8

1. Consider the action of $D_4 = \{e, r, r^2, r^3, s, sr, sr^2, sr^3\}$ on $R(4) = \{1, -1, i, -i\}$.

   (a) Determine whether this action is effective, transitive or free.

   (b) Find the isotropy group of i.

2. Consider the actions $\mathbb{R} \overset{\varphi}{\curvearrowright} \mathbb{R}$ and $\mathbb{R} \overset{\psi}{\curvearrowright} \mathbb{C}$ of the group $(\mathbb{R}, +)$ on the sets $\mathbb{R}$ and $\mathbb{C}$ given by

   $$\varphi_t(x) = x + t \qquad \text{and} \qquad \psi_t(z) = e^{it} z$$

   for all $t, x \in \mathbb{R}$ and all $z \in \mathbb{C}$.

   (a) Determine whether these actions are effective, transitive or free.

   (b) Find all isotropy subgroups and all fixed points of these actions.

   (c) Give an example of an $\mathbb{R}$-equivariant map $T : \mathbb{R} \to \mathbb{C}$.