Algebraic and Geometric Methods in Engineering and Physics

Homework 2

Due on September 24

1. If G and H are groups then it is easy to see that $G \times H$ with the binary operation

$$(g_1, h_1) \cdot (g_2, h_2) = (g_1 g_2, h_1 h_2)$$

is also a group. Show that:

(a) $\mathbb{Z}_2 \times \mathbb{Z}_2$ is not cyclic, and therefore is not isomorphic to $\mathbb{Z}_4$.

(b) $\mathbb{Z}_2 \times \mathbb{Z}_2$ and $\mathbb{Z}_4$ are the only groups of order 4 (up to isomorphism).

(c) All groups of order ≤ 5 are abelian.

2. Consider the **unitary group**

$$U_n = \{A \in GL_n(\mathbb{C}) : A^* A = I\},$$

and the **special unitary group**

$$SU_n = \{A \in U_n : \det A = 1\}.$$

Prove that SU_n is a normal subgroup of U_n, and also that $U_n / SU_n \cong S^1$.