

Algebraic and Geometric Methods in Engineering and Physics

Homework 12

Due on December 18

1. Consider the operation

$$(x, y) \cdot (z, w) = (yz + x, yw)$$

defined on the open set

$$H = \{(x, y) \in \mathbb{R}^2 : y > 0\}.$$

(a) Show that $(H, \cdot)$ is a Lie group.

(b) Show that the derivative of the left translation map $L_{(x,y)} : H \rightarrow H$ at a given point $(z, w) \in H$ is represented by the matrix

$$(dL_{(x,y)})_{(z,w)} = \begin{pmatrix} y & 0 \\ 0 & y \end{pmatrix}.$$

Conclude that the left-invariant vector field $X^V \in \mathfrak{X}(H)$ determined by the vector

$$V = V^1 \frac{\partial}{\partial x} + V^2 \frac{\partial}{\partial y} \in \mathfrak{h} \equiv T_{(0,1)}H$$

is given by

$$X_{(x,y)}^V = yV^1 \frac{\partial}{\partial x} + yV^2 \frac{\partial}{\partial y}.$$

(c) Given $V, W \in \mathfrak{h}$, compute $[V, W]$.

(d) Determine the flow of the vector field X^V .