

# Algebraic and Geometric Methods in Engineering and Physics

## Homework 8

*Due on November 30*

1. Compute the characters of the representations of the group  $D_4 \equiv \{e, r, r^2, r^3, s, sr, sr^2, sr^3\}$  defined as follows:

(a)  $\varphi_r^{(1)} = \varphi_s^{(1)} = 1;$

(b)  $\varphi_r^{(2)} = 1, \varphi_s^{(2)} = -1;$

(c)  $\varphi_r^{(3)} = -1, \varphi_s^{(3)} = 1;$

(d)  $\varphi_r^{(4)} = \varphi_s^{(4)} = -1;$

(e)  $\varphi_r^{(5)} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \varphi_s^{(5)} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix};$

(f)  $\psi_r = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \psi_s = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$

2. Using the characters computed in the previous question, show that:

(a)  $\varphi^{(1)}, \varphi^{(2)}, \varphi^{(3)}, \varphi^{(4)}$  and  $\varphi^{(5)}$  are irreducible;

(b)  $\psi \sim \varphi^{(1)} \oplus \varphi^{(3)} \oplus \varphi^{(5)}.$