

Algebraic and Geometric Methods in Engineering and Physics

Homework 7

Due on November 23

1. Consider the dihedral group $D_4 \equiv \{e, r, r^2, r^3, s, sr, sr^2, sr^3\}$.

(a) Show that

$$\varphi_r = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}, \quad \varphi_s = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

defines a unitary representation φ of D_4 on \mathbb{C}^4 .

(b) Decompose this representation into a direct sum of irreducible representations, and write the corresponding block diagonal matrices for all elements in D_4 .