

# Algebraic and Geometric Methods in Engineering and Physics

## Homework 6

*Due on November 2*

1. A molecule of methane ( $CH_4$ ) consists of 4 hydrogen atoms placed at the vertices of a tetrahedron, bound to a central carbon atom. Hydrogen has three naturally occurring isotopes, namely protium (or hydrogen-1), deuterium (or hydrogen-2) and tritium (or hydrogen-3); carbon also has three naturally occurring isotopes, usually called carbon-12, carbon-13 and carbon-14. How many different kinds of molecules of methane are there? (**Hint:** The group of rotations that preserve the tetrahedron has order 12; besides the more obvious rotations, there are 3 rotations by  $180^\circ$  about the axes through the middle points of non-intersecting edges).

2. Show that

$$\varphi_r = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \varphi_s = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

defines a unitary representation  $\varphi : D_4 \equiv \{e, r, r^2, r^3, s, sr, sr^2, sr^3\} \rightarrow GL_2(\mathbb{C})$ . What are the invariant subspaces of this representation?