Algebraic and Geometric Methods in Engineering and Physics

Homework 2

Due on October 4

1. Consider the complex matrices

$$E = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad I = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}; \quad J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; \quad K = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}.$$

   (a) Show that $Q = \{E, -E, I, -I, J, -J, K, -K\}$ is a nonabelian group under matrix multiplication (**quaternionic group**).

   (b) Find all subgroups $H \subset Q$, and show that they are all normal.

   (c) Identify the quotient groups Q/H for all subgroups $H \subset Q$.

2. Consider the **unitary group**

$$U_n = \{A \in GL_n(\mathbb{C}) : A^* A = I\},$$

   and the **special unitary group**

$$SU_n = \{A \in U_n : \det A = 1\}.$$

   Prove that SU_n is a normal subgroup of U_n, and also that $U_n/SU_n \cong S^1$.