Algebraic and Geometric Methods in Engineering and Physics

Homework 11

Due on December 21

1. Consider the two charts (U, φ) and (V, ψ) for $M = \mathbb{R}^2$ given by

$$U = \mathbb{R}^2, \qquad \varphi(x, y) = (x, y)$$

and

$$\psi(V) = \mathbb{R}^+ \times (-\pi, \pi), \qquad \psi^{-1}(r, \theta) = (r\cos\theta, r\sin\theta).$$

   (a) Show that these charts are C^∞-compatible.

   (b) Write the vectors

$$\left(\frac{\partial}{\partial r}\right)_{(1,1)} \qquad \text{and} \qquad \left(\frac{\partial}{\partial \theta}\right)_{(1,1)}$$

   as linear combinations of the vectors

$$\left(\frac{\partial}{\partial x}\right)_{(1,1)} \qquad \text{and} \qquad \left(\frac{\partial}{\partial y}\right)_{(1,1)}.$$