Lie Groups and Lie Algebras

Homework 9

Due on December 3

1. Show that the center of a Lie algebra is contained in any of its Cartan subalgebras.

2. (a) Show that if $\mathfrak{g}$ is the Lie algebra of a compact Lie group then its Killing form is negative semidefinite.

   (b) Show that if in addition $\mathfrak{g}$ is simple then the Killing form is negative definite.

3. Find the Cartan matrix for $\mathfrak{sl}(n, \mathbb{C})$, and check that the corresponding Dynkin diagram is A_{n-1}.