Differential Geometry of Curves and Surfaces

Homework 13

Due on December 19

1. A given compact surface S with Gauss curvature K can be decomposed into finitely many hexagons (that is, images by some parameterization of Euclidean hexagons) whose intersections are precisely a common edge, a common vertex or empty, in such a way that exactly three edges meet at each vertex. Compute $\int_S K$.

2. A triangulation of a compact surface with boundary is defined similarly to a triangulation of a compact surface, but with the added condition that the intersection of each triangle with the boundary is either an edge, a vertex, or empty. Prove that if S is an oriented compact surface with boundary then

$$\int_S K + \int_{\partial S} \kappa_g(s) ds = 2\pi\chi(S),$$

where the Euler characteristic is defined in the same way as for compact surfaces, and the orientation of ∂S is the induced orientation.