Differential Geometry of Curves and Surfaces

Homework 11

Due on December 5

1. Recall that the the first fundamental form for the sphere $S^2 \subset \mathbb{R}^3$ is written in spherical coordinates (θ, φ) as
$$ds^2 = d\theta^2 + \sin^2\theta d\varphi^2.$$
   (a) Compute the Christoffel symbols from the coefficients of the first fundamental form and check that the geodesics equations are
$$\begin{cases} \ddot{\theta} - \sin\theta\cos\theta\,\dot{\varphi}^2 = 0 \\ \\ \ddot{\varphi} + 2\,\dfrac{\cos\theta}{\sin\theta}\,\dot{\theta}\,\dot{\varphi} = 0 \end{cases}.$$
   (b) Prove that the distance between the points with coordinates $(\frac{\pi}{4}, 0)$ and $(\frac{\pi}{4}, \frac{\pi}{2})$ is smaller than $\frac{\pi\sqrt{2}}{4}$.

2. Consider the Riemannian metric on $\mathbb{R}^2$ given by
$$ds^2 = du^2 + G(u,v)dv^2.$$
   Prove that the curve $c(t) = (t, v_0)$ (with v_0 constant) is a geodesic by showing that:
   (a) $c(t)$ is a solution of the geodesic equations.
   (b) $c(t)$ minimizes the distance between any two points of the form (u_0, v_0) and (u_1, v_0).