Differential Geometry of Curves and Surfaces

Homework 9

Due on November 30

1. Recall that the **hyperbolic plane** is the open domain $H = \{(x,y) \in \mathbb{R}^2 : y > 0\}$ with the Riemannian metric
$$ds^2 = \frac{1}{y^2}\left(dx^2 + dy^2\right).$$
Show that:
   (a) This Riemannian metric can be written as
   $$ds^2 = \left(\theta^1\right)^2 + \left(\theta^2\right)^2,$$
   where
   $$\theta^1 = \frac{1}{y}dx, \qquad \theta^2 = \frac{1}{y}dy.$$
   (b) The orthonornormal frame $\{\mathbf{e}_1, \mathbf{e}_2\}$ dual to $\{\theta^1, \theta^2\}$ is given by
   $$\mathbf{e}_1 = y\frac{\partial}{\partial x}, \qquad \mathbf{e}_2 = y\frac{\partial}{\partial y}.$$
   (c) The connection form associated to $\{\theta^1, \theta^2\}$ is
   $${\omega_2}^1 = -\frac{1}{y}dx.$$
   (d) The Gauss curvature of the hyperbolic plane is $K(x,y) = -1$.
   (e) The vector field
   $$\mathbf{V}(t) = \cos t\frac{\partial}{\partial x} - \sin t\frac{\partial}{\partial y}$$
   is parallel along the curve given by $(x(t), y(t)) = (t, 1)$. Is this curve turning left or turning right?