Differential Geometry of Curves and Surfaces

Homework 8

Due on November 24

1. Consider the surface of revolution S generated by the curve $\mathbf{c} : (s_0, s_1) \to \mathbb{R}^2$ given by $\mathbf{c}(s) = (f(s), g(s))$ (with $f(s) > 0$), where s is the arclength:

$$\big(f'(s)\big)^2 + \big(g'(s)\big)^2 = 1.$$

A parameterization of this surface is $\mathbf{g} : (s_0, s_1) \times (0, 2\pi) \to S$ given by

$$\mathbf{g}(s, \varphi) = (f(s)\cos\varphi, f(s)\sin\varphi, g(s)).$$

Using the method of orthonormal frames, show that:

(a) The Gauss curvature of S is

$$K(s, \varphi) = -\frac{f''(s)}{f(s)}.$$

(b) The mean curvature of S is

$$H(s, \varphi) = \frac{g'(s)}{2f(s)} - \frac{f''(s)}{2g'(s)}.$$

(c) If S is flat then the image of $\mathbf{c}$ is a line segment (and so S is a subset of a cone, a cylinder or a plane).