Differential Geometry of Curves and Surfaces

Homework 4

Due on October 13

1. Show that $\mathbf{g} : \mathbb{R} \times (-\pi, \pi) \to \mathbb{R}^{\mathbf{3}}$ given by

$$\mathbf{g}(u, \varphi) = (\cosh u \cos \varphi, \cosh u \sin \varphi, \sinh u)$$

is a parameterization of the 2-dimensional manifold

$$M = \{(x, y, z) \in \mathbb{R}^4 : x^2 + y^2 = z^2 + 1\},$$

and find $T^{\perp}_{(2,1,2)} M$.

2. Because $\binom{3}{1} = \binom{3}{2} = 3$, it is possible to identify $\mathbb{R}^3$ both with $\Lambda^1\left(\mathbb{R}^3\right)$ and with $\Lambda^2\left(\mathbb{R}^3\right)$: if $\mathbf{v} \in \mathbb{R}^3$, we define

$$\omega_{\mathbf{v}} = v^1 dx + v^2 dy + v^3 dz$$

and

$$\Omega_{\mathbf{v}} = v^1 dy \wedge dz + v^2 dz \wedge dx + v^3 dx \wedge dy.$$

Show that:

(a) $\omega_{\mathbf{v}}(\mathbf{w}) = \mathbf{v} \cdot \mathbf{w}$;

(b) $\Omega_{\mathbf{u}}(\mathbf{v}, \mathbf{w}) = \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w})$;

(c) $\omega_{\mathbf{v}} \wedge \omega_{\mathbf{w}} = \Omega_{\mathbf{v} \times \mathbf{w}}$;

(d) $\omega_{\mathbf{v}} \wedge \Omega_{\mathbf{w}} = (\mathbf{v} \cdot \mathbf{w})\, dx \wedge dy \wedge dz$.