Differential Geometry of Curves and Surfaces

Homework 3

Due on October 6

Use a symbolic computation system, such as Mathematica, to solve this problem

1. Consider the simple closed regular space curve $\mathbf{c} : [0, 2\pi] \to \mathbb{R}^3$ given by

$$\mathbf{c}(t) = ((2 + \cos(3t))\cos(2t), (2 + \cos(3t))\sin(2t), \sin(3t))).$$

   (a) Obtain a plot of this curve. Based on this plot, is this curve the unknot?

   (b) Obtain a numerical estimate of its total curvature, and check that it is bigger than 4π.

2. Consider the sets

$$S^3 = \{(x, y, z, w) \in \mathbb{R}^4 : x^2 + y^2 + z^2 + w^2 = 1\}$$

   and

$$T^2 = \{(x, y, z, w) \in \mathbb{R}^4 : x^2 + y^2 = 1/2 \text{ and } z^2 + w^2 = 1/2\}.$$

   (a) Show that both sets are manifolds, and compute their dimensions.

   (b) Show that if $\mathbf{c} : [0, 1] \to S^3$ is a continuous curve such that $\mathbf{c}(0) = (1, 0, 0, 0)$ and $\mathbf{c}(1) = (0, 0, 0, 1)$ then there must exist $t \in (0, 1)$ such that $\mathbf{c}(t) \in T^2 \subset S^3$.