

Partial Differential Equations

Homework 8

Due on April 30

1. Show that if $f \in C^\infty(\Omega)$ and $T \in \mathcal{D}'(\Omega)$, with $\Omega \subset \mathbb{R}^N$ an open set, then

$$\partial_i(fT) = (\partial_i f)T + f\partial_i T.$$

2. Let $H \in L^1_{\text{loc}}(\mathbb{R})$ be Heaviside's function,

$$H(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x < 0 \end{cases}.$$

Use $H = H^2 = H^3$ to prove that it is impossible to define a product (that is, a commutative bilinear operation) on $\mathcal{D}'(\mathbb{R})$ which generalizes the multiplication of functions in $L^1_{\text{loc}}(\mathbb{R})$ and satisfies Leibnitz's rule for the derivative of a product.