

Partial Differential Equations

Homework 1

Due on February 27

1. Consider the initial value problem for the non-homogeneous transport equation given by

$$\begin{cases} u_t + u_x = x \\ u(x, 0) = 0 \end{cases},$$

where $u : \mathbb{R}^2 \rightarrow \mathbb{R}$ is assumed to be of class C^1 .

- (a) Write the unique solution to this problem explicitly.
- (b) Plot the graph of $u(x, t)$ as a function of x for $t = 0$, $t = 1$ and $t = 2$ (feel free to use your favorite graphing software). Can you interpret what is going on?