Stability of laminar flows in the presence of magnetic fields and possible applications

Paolo Falsaperla

Joint work with G. Mulone, A. Giacobbe

University of Catania
Dipartimento di Matematica e Informatica

PDE's and Biomedical Applications

Lisbon - December 4-6, 2014

Introduction

We study the stability of stationary *laminar flows*.

Introduction

We study the stability of stationary *laminar flows*.

Laminar flows occur when a fluid flows in *parallel layers*, such as parallel planes (plane flow) or concentric cylinders (pipe flow).

Introduction

We study the stability of stationary *laminar flows*.

Laminar flows occur when a fluid flows in *parallel layers*, such as parallel planes (plane flow) or concentric cylinders (pipe flow).

In general, above a critical velocity of the fluid, the laminar flow becomes unstable giving rise to a mixing of the fluid of different layers (turbulent flow).

Introduction

We study the stability of stationary *laminar flows*.

Laminar flows occur when a fluid flows in *parallel layers*, such as parallel planes (plane flow) or concentric cylinders (pipe flow).

In general, above a critical velocity of the fluid, the laminar flow becomes unstable giving rise to a mixing of the fluid of different layers (turbulent flow).

It is important to be able to model and predict such transition, since many property of the flow depend on it:

- Mixing of fluid components
- Reduced flow and energy dissipation
- Mechanical effects on the boundaries
- ...

Physical effects

The instability threshold can be affected by many physical causes, in this work we examine the effect of

Physical effects

The instability threshold can be affected by many physical causes, in this work we examine the effect of

- a superimposed magnetic field (for electrically conducting fluids).

Physical effects

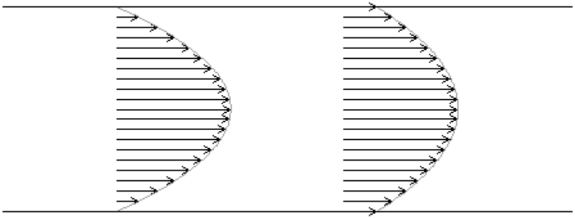
The instability threshold can be affected by many physical causes, in this work we examine the effect of

- a superimposed magnetic field (for electrically conducting fluids).
- finite slip boundary conditions (graphic).

Physical effects

The instability threshold can be affected by many physical causes, in this work we examine the effect of

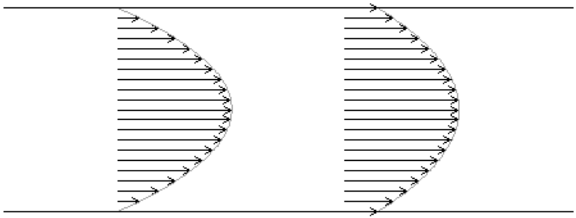
- a superimposed magnetic field (for electrically conducting fluids).
- finite slip boundary conditions (graphic).



Physical effects

The instability threshold can be affected by many physical causes, in this work we examine the effect of

- a superimposed magnetic field (for electrically conducting fluids).
- finite slip boundary conditions (graphic).



We will also discuss some literature on the effects of magnetic fields on blood flow.

The system

We will study a *plane laminar flow*, but similar techniques can be used to study

The system

We will study a *plane laminar flow*, but similar techniques can be used to study

- Flows in tubes

The system

We will study a *plane laminar flow*, but similar techniques can be used to study

- Flows in tubes
- Flows between coaxial cylinders

The system

We will study a *plane laminar flow*, but similar techniques can be used to study

- Flows in tubes
- Flows between coaxial cylinders

Laminar flows subject to parallel magnetic fields appear in many physical systems. Some meaningful examples are

- MHD plasma confinement
- liquid-metal cooling of nuclear reactors
- electromagnetic casting

The system

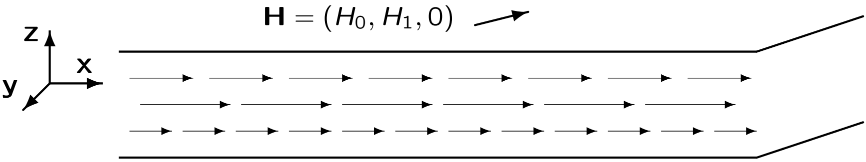
We consider an infinite horizontal layer of a homogeneous, viscous, electrically conducting fluid, permeated by an imposed magnetic field parallel to the layer.

The system

We consider an infinite horizontal layer of a homogeneous, viscous, electrically conducting fluid, permeated by an imposed magnetic field parallel to the layer.
Let $d > 0$, $\Omega_d = \mathbb{R}^2 \times (-d, d)$ and $Oxyz$ be a Cartesian frame of reference with unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$, respectively.

The system

We consider an infinite horizontal layer of a homogeneous, viscous, electrically conducting fluid, permeated by an imposed magnetic field parallel to the layer.
Let $d > 0$, $\Omega_d = \mathbb{R}^2 \times (-d, d)$ and $Oxyz$ be a Cartesian frame of reference with unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$, respectively.
We will study laminar flows, subject to a constant external magnetic field.



Motivations, literature

Stability of this magnetohydrodynamic systems was investigated since the beginning of the century.

Motivations, literature

Stability of this magnetohydrodynamic systems was investigated since the beginning of the century.

We recall here also some works from prof. Rionero on asymptotic and nonlinear stability of such systems.

Motivations, literature

Stability of this magnetohydrodynamic systems was investigated since the beginning of the century.

We recall here also some works from prof. Rionero on asymptotic and nonlinear stability of such systems.

- Sulla stabilità asintotica in media in MHD, *Ann. Mat. P. Appl.* (1967)

Motivations, literature

Stability of this magnetohydrodynamic systems was investigated since the beginning of the century.

We recall here also some works from prof. Rionero on asymptotic and nonlinear stability of such systems.

- Sulla stabilità asintotica in media in MHD, *Ann. Mat. P. Appl.* (1967)
- Sulla stabilità asintotica in media in MHD non isoterma, *Ricerche Mat.* (1967)

Motivations, literature

Stability of this magnetohydrodynamic systems was investigated since the beginning of the century.

We recall here also some works from prof. Rionero on asymptotic and nonlinear stability of such systems.

- Sulla stabilità asintotica in media in MHD, *Ann. Mat. P. Appl.* (1967)
- Sulla stabilità asintotica in media in MHD non isoterma, *Ricerche Mat.* (1967)
- Metodi variazionali per la stabilità asintotica in media in MHD, *Ann. Mat. Pura Appl.* (1968)

Motivations, literature

Stability of this magnetohydrodynamic systems was investigated since the beginning of the century.

We recall here also some works from prof. Rionero on asymptotic and nonlinear stability of such systems.

- Sulla stabilità asintotica in media in MHD, *Ann. Mat. P. Appl.* (1967)
- Sulla stabilità asintotica in media in MHD non isoterma, *Ricerche Mat.* (1967)
- Metodi variazionali per la stabilità asintotica in media in MHD, *Ann. Mat. Pura Appl.* (1968)
- Sulla stabilità MHD non lineare asintotica in media con vari tipi di condizioni al contorno, *Ricerche di Mat.* (1968)

Motivations, literature

Stability of this magnetohydrodynamic systems was investigated since the beginning of the century.

We recall here also some works from prof. Rionero on asymptotic and nonlinear stability of such systems.

- Sulla stabilità asintotica in media in MHD, *Ann. Mat. P. Appl.* (1967)
- Sulla stabilità asintotica in media in MHD non isoterma, *Ricerche Mat.* (1967)
- Metodi variazionali per la stabilità asintotica in media in MHD, *Ann. Mat. Pura Appl.* (1968)
- Sulla stabilità MHD non lineare asintotica in media con vari tipi di condizioni al contorno, *Ricerche di Mat.* (1968)
- On magnetohydrodynamic stability, *Quaderni di Matematica* (1997)

Equations

The fluid is modeled by the following equations (see e.g. Chandrasekhar, 1961):

Equations

The fluid is modeled by the following equations (see e.g. Chandrasekhar, 1961):

$$\begin{cases} \mathbf{U}_t + \mathbf{U}\cdot\nabla\mathbf{U} - \frac{\mu}{\rho_0}\mathbf{H}\cdot\nabla\mathbf{H} = -\nabla(\frac{p_1}{\rho_0} + \frac{\mu H^2}{2\rho_0}) + \nu\Delta\mathbf{U} \\ \mathbf{H}_t + \mathbf{U}\cdot\nabla\mathbf{H} - \mathbf{H}\cdot\nabla\mathbf{U} = \eta\Delta\mathbf{H} \\ \nabla\cdot\mathbf{U} = 0, \quad \nabla\cdot\mathbf{H} = 0\,, \end{cases}$$

Equations

The fluid is modeled by the following equations (see e.g. Chandrasekhar, 1961):

$$\begin{cases} \mathbf{U}_t + \mathbf{U} \cdot \nabla \mathbf{U} - \frac{\mu}{\rho_0} \mathbf{H} \cdot \nabla \mathbf{H} = -\nabla \left(\frac{p_1}{\rho_0} + \frac{\mu H^2}{2\rho_0} \right) + \nu \Delta \mathbf{U} \\ \mathbf{H}_t + \mathbf{U} \cdot \nabla \mathbf{H} - \mathbf{H} \cdot \nabla \mathbf{U} = \eta \Delta \mathbf{H} \\ \nabla \cdot \mathbf{U} = 0, \quad \nabla \cdot \mathbf{H} = 0 \,, \end{cases}$$

where $\mathbf{U}, \mathbf{H}$ and p_1 are the velocity, magnetic and pressure fields, respectively; ν is the kinematic viscosity, η is the magnetic viscosity ($\eta = \frac{1}{\mu\sigma}$) with μ the magnetic permeability and σ the electrical conductivity, ρ_0 is the constant density.

The stationary equations

The stationary equations are

$$\begin{cases} \mathbf{U}\cdot\nabla\mathbf{U} - \frac{\mu}{\rho_0}\mathbf{H}\cdot\nabla\mathbf{H} = -\nabla\left(\frac{p_1}{\rho_0} + \frac{\mu H^2}{2\rho_0}\right) + \nu\Delta\mathbf{U} \\ \mathbf{U}\cdot\nabla\mathbf{H} = \mathbf{H}\cdot\nabla\mathbf{U} + \eta\Delta\mathbf{H} \\ \nabla\cdot\mathbf{U} = 0, \quad \nabla\cdot\mathbf{H} = 0. \end{cases}$$

The stationary equations

The stationary equations are

$$\begin{cases} \mathbf{U} \cdot \nabla \mathbf{U} - \frac{\mu}{\rho_0} \mathbf{H} \cdot \nabla \mathbf{H} = -\nabla \left(\frac{p_1}{\rho_0} + \frac{\mu H^2}{2\rho_0} \right) + \nu \Delta \mathbf{U} \\ \mathbf{U} \cdot \nabla \mathbf{H} = \mathbf{H} \cdot \nabla \mathbf{U} + \eta \Delta \mathbf{H} \\ \nabla \cdot \mathbf{U} = 0, \quad \nabla \cdot \mathbf{H} = 0. \end{cases}$$

And we look in particular for laminar solutions:
$\mathbf{U}(z)$, $\mathbf{U} \cdot k = 0$, so $\mathbf{U} = (U(z), V(z), 0)$, and also $\mathbf{H} = \text{const.}$

The stationary equations

The stationary equations are

$$\begin{cases} \mathbf{U}\cdot\nabla\mathbf{U} - \frac{\mu}{\rho_0}\mathbf{H}\cdot\nabla\mathbf{H} = -\nabla(\frac{p_1}{\rho_0} + \frac{\mu H^2}{2\rho_0}) + \nu\Delta\mathbf{U} \\ \mathbf{U}\cdot\nabla\mathbf{H} = \mathbf{H}\cdot\nabla\mathbf{U} + \eta\Delta\mathbf{H} \\ \nabla\cdot\mathbf{U} = 0, \quad \nabla\cdot\mathbf{H} = 0. \end{cases}$$

And we look in particular for laminar solutions:
$\mathbf{U}(z)$, $\mathbf{U}\cdot k = 0$, so $\mathbf{U} = (U(z), V(z), 0)$, and also $\mathbf{H} = \text{const}$.

It can be proved that (unless $\mathbf{U}\equiv 0$) this implies also $\mathbf{H}\cdot z = 0$, that is $\mathbf{H}$ must be coplanar to the flow.

Boundary conditions

We assume that the boundaries are stationary, and that some "slip" of the flow is present. We assume also that the medium surrounding the fluid is electrically non-conducting.

Boundary conditions

We assume that the boundaries are stationary, and that some "slip" of the flow is present. We assume also that the medium surrounding the fluid is electrically non-conducting.

$$\begin{cases} \mathbf{U}(x,y,-d) = \mathbf{U}(x,y,d) = 0 \\ U(d) = -\lambda U_z(d), \quad U(-d) = \lambda U_z(-d) \\ V(d) = -\lambda V_z(d), \quad V(-d) = \lambda V_z(-d) \\ \mathbf{H}(x,y,-d) = H_0\mathbf{i} + H_1\mathbf{j} \quad \mathbf{H}(x,y,d) = H_0\mathbf{i} + H_1\mathbf{j} \end{cases}$$

where $H_0, H_1 \in \mathbb{R}$, $H_0 \neq 0$.

Boundary conditions

We assume that the boundaries are stationary, and that some "slip" of the flow is present. We assume also that the medium surrounding the fluid is electrically non-conducting.

$$\begin{cases} \mathbf{U}(x,y,-d) = \mathbf{U}(x,y,d) = 0 \\ U(d) = -\lambda U_z(d), \quad U(-d) = \lambda U_z(-d) \\ V(d) = -\lambda V_z(d), \quad V(-d) = \lambda V_z(-d) \\ \mathbf{H}(x,y,-d) = H_0\mathbf{i} + H_1\mathbf{j} \quad \mathbf{H}(x,y,d) = H_0\mathbf{i} + H_1\mathbf{j} \end{cases}$$

where $H_0, H_1 \in \mathbb{R}$, $H_0 \neq 0$.
The static equations admit then the laminar solutions

$$\begin{cases} \mathbf{H} = H_0\mathbf{i} + H_1\mathbf{j} \\ U(z) = \bar{U}_0\left(1 - \frac{z^2}{d^2+2\lambda d}\right), V(z) = 0 \\ p_1 = -k\rho_0 x + p_0 \end{cases}$$

with k and $p_0 \in \mathbb{R}$, and $\bar{U}_0 = \frac{k(d^2+2\lambda d)}{2\nu}$.

Boundary conditions

We assume that the boundaries are stationary, and that some "slip" of the flow is present. We assume also that the medium surrounding the fluid is electrically non-conducting.

$$\begin{cases} \mathbf{U}(x,y,-d) = \mathbf{U}(x,y,d) = 0 \\ U(d) = -\lambda U_z(d), \quad U(-d) = \lambda U_z(-d) \\ V(d) = -\lambda V_z(d), \quad V(-d) = \lambda V_z(-d) \\ \mathbf{H}(x,y,-d) = H_0\mathbf{i} + H_1\mathbf{j} \quad \mathbf{H}(x,y,d) = H_0\mathbf{i} + H_1\mathbf{j} \end{cases}$$

where $H_0, H_1 \in \mathbb{R}$, $H_0 \neq 0$.
The static equations admit then the laminar solutions

$$\begin{cases} \mathbf{H} = H_0\mathbf{i} + H_1\mathbf{j} \\ U(z) = \bar{U}_0\left(1 - \frac{z^2}{d^2+2\lambda d}\right), V(z) = 0 \\ p_1 = -k\rho_0 x + p_0 \end{cases}$$

with k and $p_0 \in \mathbb{R}$, and $\bar{U}_0 = \frac{k(d^2+2\lambda d)}{2\nu}$. This general form includes the rigid case (for $\lambda = 0$). Note that we assume here the same slip length on both boundaries.

Equations for the perturbations

The non-dimensional equations of a perturbation $(\mathbf{u}, \mathbf{h})$ to the previous solution are

$$\begin{cases} \mathbf{u}_t + \mathbf{u}\cdot\nabla\mathbf{u} + wU'(z)\mathbf{i} + U(z)\mathbf{u}_x = \\ \qquad A_m[H_0\mathbf{h}_x + H_1\mathbf{h}_y + \mathbf{h}\cdot\nabla\mathbf{h}] - \nabla\lambda + \frac{1}{R_e}\Delta\mathbf{u} \\ \mathbf{h}_t + \mathbf{u}\cdot\nabla\mathbf{h} + U(z)\mathbf{h}_x = H_0\mathbf{u}_x + H_1\mathbf{u}_y + hU'(z)\mathbf{i} + \mathbf{h}\cdot\nabla\mathbf{u} + \frac{1}{R_m}\Delta\mathbf{h} \\ \nabla\cdot\mathbf{u} = 0, \quad \nabla\cdot\mathbf{h} = 0 \end{cases}$$

Equations for the perturbations

The non-dimensional equations of a perturbation $(\mathbf{u}, \mathbf{h})$ to the previous solution are

$$\begin{cases} \mathbf{u}_t + \mathbf{u}\cdot\nabla\mathbf{u} + wU'(z)\mathbf{i} + U(z)\mathbf{u}_x = \\ \qquad A_m[H_0\mathbf{h}_x + H_1\mathbf{h}_y + \mathbf{h}\cdot\nabla\mathbf{h}] - \nabla\lambda + \frac{1}{R_e}\Delta\mathbf{u} \\ \mathbf{h}_t + \mathbf{u}\cdot\nabla\mathbf{h} + U(z)\mathbf{h}_x = H_0\mathbf{u}_x + H_1\mathbf{u}_y + hU'(z)\mathbf{i} + \mathbf{h}\cdot\nabla\mathbf{u} + \frac{1}{R_m}\Delta\mathbf{h} \\ \nabla\cdot\mathbf{u} = 0, \quad \nabla\cdot\mathbf{h} = 0 \end{cases}$$

in $\mathbb{R}^2 \times (-1,1) \times (0,+\infty)$, where
$\mathbf{h} = (h_1, h_2, h)$, $\mathbf{u} = (u, v, w)$, $\lambda = \frac{p}{\rho_0} + \frac{A_m|\mathbf{H}+\mathbf{h}|^2}{2}$.

Equations for the perturbations

The non-dimensional equations of a perturbation $(\mathbf{u},\mathbf{h})$ to the previous solution are

$$\begin{cases} \mathbf{u}_t + \mathbf{u}\cdot\nabla\mathbf{u} + wU'(z)\mathbf{i} + U(z)\mathbf{u}_x = \\ \qquad A_m[H_0\mathbf{h}_x + H_1\mathbf{h}_y + \mathbf{h}\cdot\nabla\mathbf{h}] - \nabla\lambda + \frac{1}{R_e}\Delta\mathbf{u} \\ \mathbf{h}_t + \mathbf{u}\cdot\nabla\mathbf{h} + U(z)\mathbf{h}_x = H_0\mathbf{u}_x + H_1\mathbf{u}_y + hU'(z)\mathbf{i} + \mathbf{h}\cdot\nabla\mathbf{u} + \frac{1}{R_m}\Delta\mathbf{h} \\ \nabla\cdot\mathbf{u} = 0, \quad \nabla\cdot\mathbf{h} = 0 \end{cases}$$

in $\mathbb{R}^2\times(-1,1)\times(0,+\infty)$, where
$\mathbf{h} = (h_1, h_2, h)$, $\mathbf{u} = (u, v, w)$, $\lambda = \frac{p}{\rho_0} + \frac{A_m|\mathbf{H}+\mathbf{h}|^2}{2}$.
R_e is the Reynolds number $R_e = \frac{\bar{U}_0 d}{\nu}$,

Equations for the perturbations

The non-dimensional equations of a perturbation $(\mathbf{u}, \mathbf{h})$ to the previous solution are

$$\begin{cases} \mathbf{u}_t + \mathbf{u}\cdot\nabla\mathbf{u} + wU'(z)\mathbf{i} + U(z)\mathbf{u}_x = \\ \qquad A_m[H_0\mathbf{h}_x + H_1\mathbf{h}_y + \mathbf{h}\cdot\nabla\mathbf{h}] - \nabla\lambda + \frac{1}{R_e}\Delta\mathbf{u} \\ \mathbf{h}_t + \mathbf{u}\cdot\nabla\mathbf{h} + U(z)\mathbf{h}_x = H_0\mathbf{u}_x + H_1\mathbf{u}_y + hU'(z)\mathbf{i} + \mathbf{h}\cdot\nabla\mathbf{u} + \frac{1}{R_m}\Delta\mathbf{h} \\ \nabla\cdot\mathbf{u} = 0, \quad \nabla\cdot\mathbf{h} = 0 \end{cases}$$

in $\mathbb{R}^2 \times (-1,1) \times (0,+\infty)$, where
$\mathbf{h} = (h_1, h_2, h)$, $\mathbf{u} = (u, v, w)$, $\lambda = \frac{p}{\rho_0} + \frac{A_m|\mathbf{H}+\mathbf{h}|^2}{2}$.
R_e is the Reynolds number $R_e = \frac{\bar{U}_0 d}{\nu}$,
R_m is the magnetic Reynolds number $R_m = \frac{\bar{U}_0 d}{\eta}$,

Equations for the perturbations

The non-dimensional equations of a perturbation $(\mathbf{u}, \mathbf{h})$ to the previous solution are

$$\begin{cases} \mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} + wU'(z)\mathbf{i} + U(z)\mathbf{u}_x = \\ \qquad A_m[H_0\mathbf{h}_x + H_1\mathbf{h}_y + \mathbf{h} \cdot \nabla \mathbf{h}] - \nabla \lambda + \frac{1}{R_e}\Delta \mathbf{u} \\ \mathbf{h}_t + \mathbf{u} \cdot \nabla \mathbf{h} + U(z)\mathbf{h}_x = H_0\mathbf{u}_x + H_1\mathbf{u}_y + hU'(z)\mathbf{i} + \mathbf{h} \cdot \nabla \mathbf{u} + \frac{1}{R_m}\Delta \mathbf{h} \\ \nabla \cdot \mathbf{u} = 0, \quad \nabla \cdot \mathbf{h} = 0 \end{cases}$$

in $\mathbb{R}^2 \times (-1, 1) \times (0, +\infty)$, where

$\mathbf{h} = (h_1, h_2, h)$, $\mathbf{u} = (u, v, w)$, $\lambda = \frac{p}{\rho_0} + \frac{A_m|\mathbf{H}+\mathbf{h}|^2}{2}$.

R_e is the Reynolds number $R_e = \frac{\bar{U}_0 d}{\nu}$,

R_m is the magnetic Reynolds number $R_m = \frac{\bar{U}_0 d}{\eta}$,

$A_m = \frac{Q^2}{R_e R_m}$ with the Chandrasekhar number Q^2 given by $\frac{\mu H^2 d^2}{\rho_0 \nu \eta}$,

Equations for the perturbations

The non-dimensional equations of a perturbation $(\mathbf{u}, \mathbf{h})$ to the previous solution are

$$\begin{cases} \mathbf{u}_t + \mathbf{u} \cdot \nabla \mathbf{u} + wU'(z)\mathbf{i} + U(z)\mathbf{u}_x = \\ \qquad A_m[H_0 \mathbf{h}_x + H_1 \mathbf{h}_y + \mathbf{h} \cdot \nabla \mathbf{h}] - \nabla \lambda + \frac{1}{R_e} \Delta \mathbf{u} \\ \mathbf{h}_t + \mathbf{u} \cdot \nabla \mathbf{h} + U(z)\mathbf{h}_x = H_0 \mathbf{u}_x + H_1 \mathbf{u}_y + hU'(z)\mathbf{i} + \mathbf{h} \cdot \nabla \mathbf{u} + \frac{1}{R_m} \Delta \mathbf{h} \\ \nabla \cdot \mathbf{u} = 0, \quad \nabla \cdot \mathbf{h} = 0 \end{cases}$$

in $\mathbb{R}^2 \times (-1, 1) \times (0, +\infty)$, where
$\mathbf{h} = (h_1, h_2, h)$, $\mathbf{u} = (u, v, w)$, $\lambda = \frac{p}{\rho_0} + \frac{A_m |\mathbf{H} + \mathbf{h}|^2}{2}$.
R_e is the Reynolds number $R_e = \frac{\bar{U}_0 d}{\nu}$,
R_m is the magnetic Reynolds number $R_m = \frac{\bar{U}_0 d}{\eta}$,
$A_m = \frac{Q^2}{R_e R_m}$ with the Chandrasekhar number Q^2 given by $\frac{\mu H^2 d^2}{\rho_0 \nu \eta}$,
$\bar{U}_0$ is a reference velocity (the maximum of U).

Linearization, periodic perturbations

For a stability study, we investigate the evolution of small perturbations, we assume also that such fields are periodic in the x, y plane.

Linearization, periodic perturbations

For a stability study, we investigate the evolution of small perturbations, we assume also that such fields are periodic in the x, y plane. In general a perturbation field $\mathbf{f} = (f_1, f_2, f_3)$ is then of the form

$$f_i = g(z) * e^{i(\alpha_1 x + \alpha_2 y - \beta t)}$$

Linearization, periodic perturbations

For a stability study, we investigate the evolution of small perturbations, we assume also that such fields are periodic in the x, y plane. In general a perturbation field $\mathbf{f} = (f_1, f_2, f_3)$ is then of the form

$$f_i = g(z) * e^{i(\alpha_1 x + \alpha_2 y - \beta t)}$$

where $2\pi/\alpha_1$, $2\pi/\alpha_2$ are wave lengths in the x and y directions, and β is the complex frequency.

Linearization, periodic perturbations

For a stability study, we investigate the evolution of small perturbations, we assume also that such fields are periodic in the x, y plane. In general a perturbation field $\mathbf{f} = (f_1, f_2, f_3)$ is then of the form

$$f_i = g(z) * e^{i(\alpha_1 x + \alpha_2 y - \beta t)}$$

where $2\pi/\alpha_1$, $2\pi/\alpha_2$ are wave lengths in the x and y directions, and β is the complex frequency.We define also $\alpha^2 = \alpha_1^2 + \alpha_2^2$, and $c = \beta/\alpha$, and assume for simplicity that the coplanar magnetic field is directed along x.

Final equations

Skipping some calculations, the final relevant equations for the system are

$$(U-c)\Psi - W = -\frac{i}{R_m\alpha}(D^2\psi - \alpha^2)\Psi$$
$$(U-c)(D^2-\alpha^2)W - WU'' - A_m(D^2-\alpha^2)\Psi = -\frac{i}{R_e\alpha}(D^2-\alpha^2)^2 W$$

Final equations

Skipping some calculations, the final relevant equations for the system are

$$(U-c)\Psi - W = -\frac{i}{R_m\alpha}(D^2\psi - \alpha^2)\Psi$$
$$(U-c)(D^2-\alpha^2)W - WU'' - A_m(D^2-\alpha^2)\Psi = -\frac{i}{R_e\alpha}(D^2-\alpha^2)^2 W$$

where Ψ and W contain the z dependency of the z-component of the perturbing fields, i.e.

$$h_3 = \Psi(z) * e^{i\cdots}, \quad w = W(z) * e^{i\cdots}$$

Final equations

Skipping some calculations, the final relevant equations for the system are

$$(U-c)\Psi - W = -\frac{i}{R_m\alpha}(D^2\psi - \alpha^2)\Psi$$
$$(U-c)(D^2-\alpha^2)W - WU'' - A_m(D^2-\alpha^2)\Psi = -\frac{i}{R_e\alpha}(D^2-\alpha^2)^2 W$$

where Ψ and W contain the z dependency of the z-component of the perturbing fields, i.e.

$$h_3 = \Psi(z) * e^{i\cdots}, \quad w = W(z) * e^{i\cdots}$$

D denotes the derivative along z.

Spectrum for simple Orr-Sommerfeld

The previous equation reduce (necessarily) to the usual Orr-Sommerfeld equations in the absence of a magnetic field.

Spectrum for simple Orr-Sommerfeld

The previous equation reduce (necessarily) to the usual Orr-Sommerfeld equations in the absence of a magnetic field.

They (and the O-S eq.) are an eigensystem in the complex frequency c, which describes solutions exponentially increasing or decreasing in time, according to the sign of the imaginary part of c.

Spectrum for simple Orr-Sommerfeld

The previous equation reduce (necessarily) to the usual Orr-Sommerfeld equations in the absence of a magnetic field.

They (and the O-S eq.) are an eigensystem in the complex frequency c, which describes solutions exponentially increasing or decreasing in time, according to the sign of the imaginary part of c.

We solved such systems by means of a Chebychev-tau scheme.

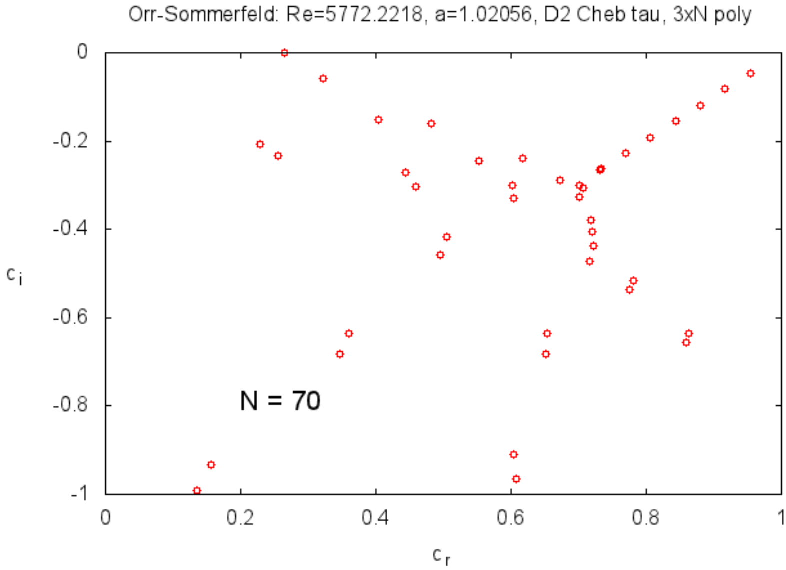
Spectrum for simple Orr-Sommerfeld

The previous equation reduce (necessarily) to the usual Orr-Sommerfeld equations in the absence of a magnetic field.

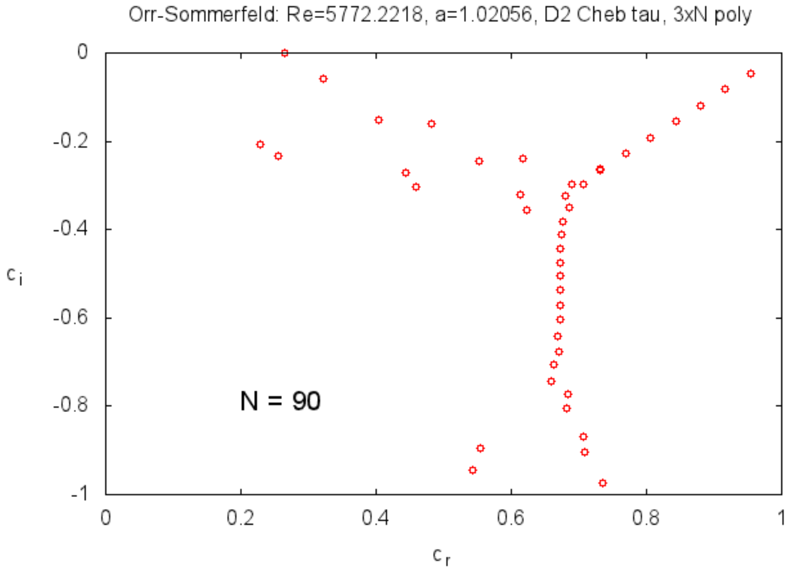
They (and the O-S eq.) are an eigensystem in the complex frequency c, which describes solutions exponentially increasing or decreasing in time, according to the sign of the imaginary part of c.

We solved such systems by means of a Chebychev-tau scheme.

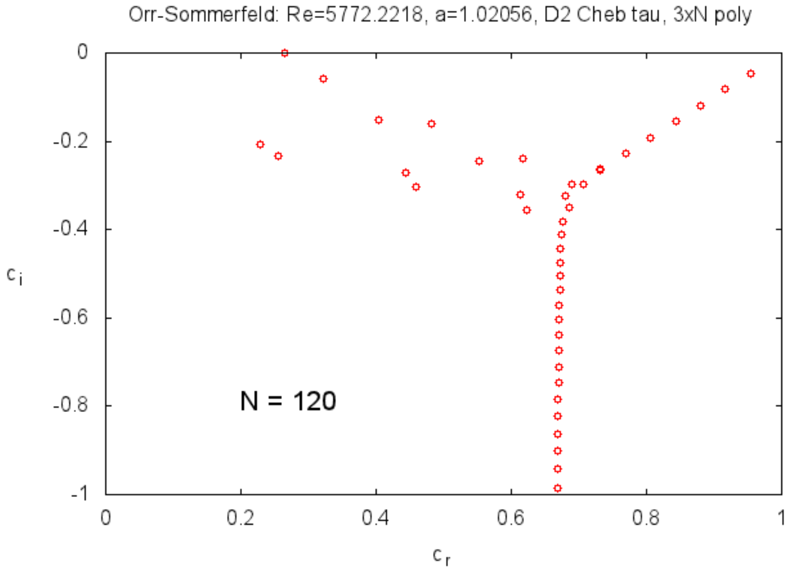
The following figures show the spectrum of c in the complex plane for the OS, for increasing number of Chebychev polynomials.



Eigenvalues close to $C_i = 0$ for $N = 70$ basis vectors per function.



Eigenvalues close to $C_i = 0$ for $N = 90$ basis vectors per function.



Eigenvalues close to $C_i = 0$ for $N = 120$ basis vectors per function.

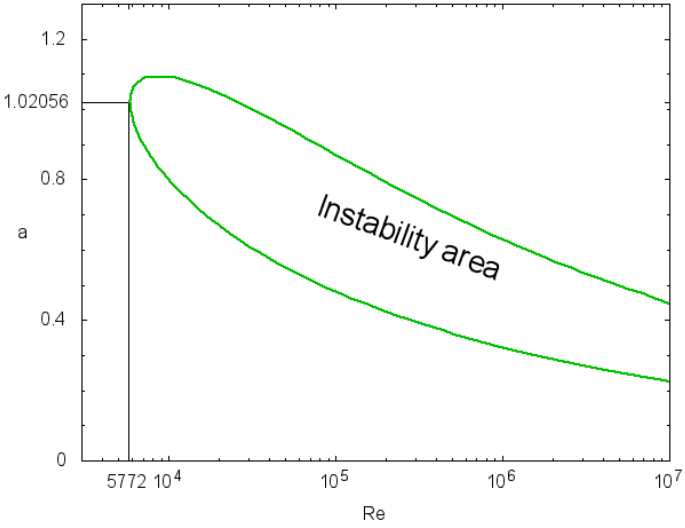
Neutral curves

A more detailed investigation looks for the neutral curves in the α, R_e plane, that is values of the couple corresponding to transition from (linear) stability to instability.

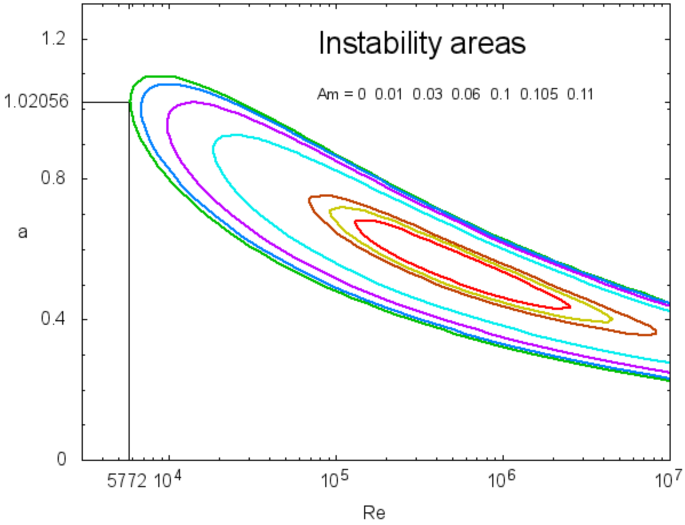
Neutral curves

A more detailed investigation looks for the neutral curves in the α, R_e plane, that is values of the couple corresponding to transition from (linear) stability to instability.

The same graphic, in the presence of an increasing magnetic field, show the stabilizing effect of H.



Critical curve in the plane of the wave number a and Reynolds number Re.



Critical curves in the plane of the wave number a and Reynolds number Re at different applied magnetic field, for a large value of the magnetic Reynolds number ($Rm = 1$).

Effect of partial slip boundaries

The effect of finite slip boundary conditions is very pronounced for this system.

Effect of partial slip boundaries

The effect of finite slip boundary conditions is very pronounced for this system.

In the medical literature slip effect are also studied, since they affect (for example) blood flow in arteries.

Effect of partial slip boundaries

The effect of finite slip boundary conditions is very pronounced for this system.

In the medical literature slip effect are also studied, since they affect (for example) blood flow in arteries.

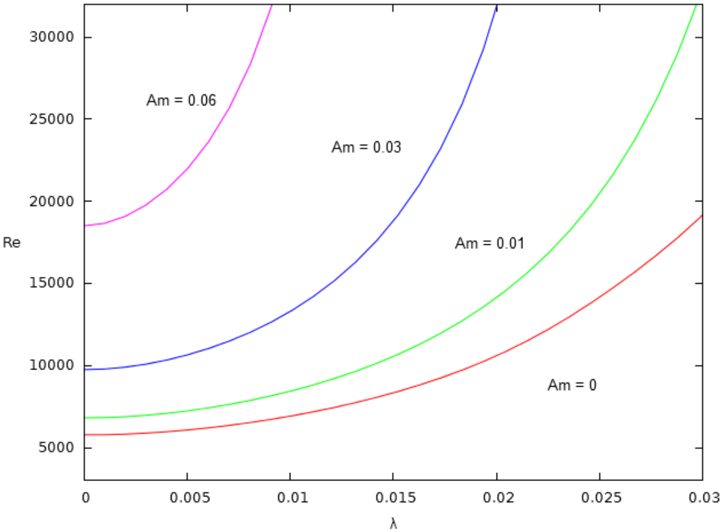
In the next graphic, we see that even slip lengths of 0.02 (in units of the half width of the layer) almost double the critical Reynolds number even in the absence of a magnetic field,

Effect of partial slip boundaries

The effect of finite slip boundary conditions is very pronounced for this system.

In the medical literature slip effect are also studied, since they affect (for example) blood flow in arteries.

In the next graphic, we see that even slip lengths of 0.02 (in units of the half width of the layer) almost double the critical Reynolds number even in the absence of a magnetic field,
and their effect is much more relevant if a magnetic field is present.



Stabilizing effect of finite slip length λ at different magnetic fields (with $Rm = 1$).

About blood viscosity

Magnetic fields reduce blood viscosity (Tao and Huang, Phys. Rev. E **84**, 011905 (2011)).

About blood viscosity

Magnetic fields reduce blood viscosity (Tao and Huang, Phys. Rev. E **84**, 011905 (2011)).

In an "in vitro" experiment, a strong magnetic field is applied for one minute along the flow direction to a sample of blood in a tube at body temperature.

About blood viscosity

Magnetic fields reduce blood viscosity (Tao and Huang, Phys. Rev. E **84**, 011905 (2011)).

In an "in vitro" experiment, a strong magnetic field is applied for one minute along the flow direction to a sample of blood in a tube at body temperature.

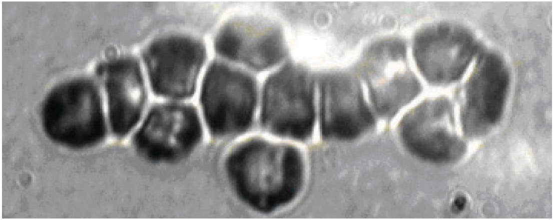
Viscosity is reduced by 33% from 7 centipoises (cp) (above healthy limits) to 4.75 cp. With no further exposure to the field, the viscosity had only risen slightly to 5.4 cp after 200 min.

About blood viscosity

The authors state that the effect is probably caused by the response of iron-rich red blood cells, which are observed to form chains or elongated aggregates after the application of the field.

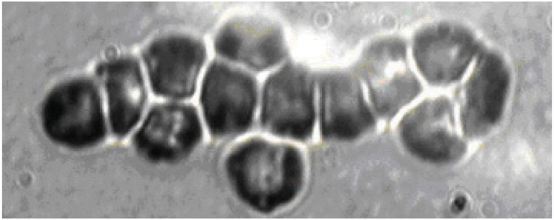
About blood viscosity

The authors state that the effect is probably caused by the response of iron-rich red blood cells, which are observed to form chains or elongated aggregates after the application of the field.



About blood viscosity

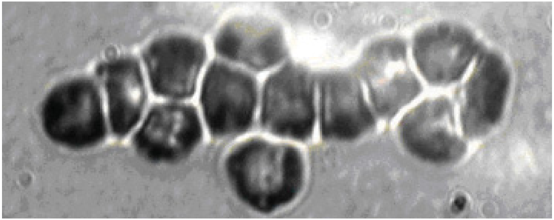
The authors state that the effect is probably caused by the response of iron-rich red blood cells, which are observed to form chains or elongated aggregates after the application of the field.



A similar effect is difficult to reproduce in a living body, due to the different orientation and diameter of the blood vessels, but this is still under investigation.

About blood viscosity

The authors state that the effect is probably caused by the response of iron-rich red blood cells, which are observed to form chains or elongated aggregates after the application of the field.



A similar effect is difficult to reproduce in a living body, due to the different orientation and diameter of the blood vessels, but this is still under investigation.

This is clearly an example of one of the many effects that can happen in a (non newtonian, and in general very complex) fluid as blood, which will be worth investigating.

THANK YOU
FOR YOUR
ATTENTION

- Rionero S., Sulla stabilità asintotica in media in magnetoidrodinamica, *Ann. Mat. Pura Appl.* **76**, (1967) 75–92.
- Rionero S., Sulla stabilità asintotica in media in magnetoidrodinamica non isoterma, *Ricerche Mat.* **16**, (1967) 250–263.
- Rionero S., (1968) Metodi variazionali per la stabilità asintotica in media in magnetoidrodinamica, *Ann. Mat. Pura Appl.*, **78**, 339–364.
- Rionero S., (1968) Sulla stabilità magnetoidrodinamica non lineare asintotica in media con vari tipi di condizioni al contorno, *Ricerche di Mat.* **17**, 64–78.
- Rionero S., (1997) On magnetohydrodynamic stability, *Quaderni di Matematica* **1**, 347–376.
- Chandrasekhar, S. *Hydrodynamic and hydromagnetic stability*. Oxford: Clarendon Press, 1961.

- G. Mulone and S. Rionero, Necessary and sufficient conditions in the magnetic Bénard problem, *Arch. Rational Mech. Anal.* **166** no. 3, 197–218, (2003).
- Tao R., Huang K., Reducing blood viscosity with magnetic fields. Phys. Rev. E **84**, 011905 (2011).