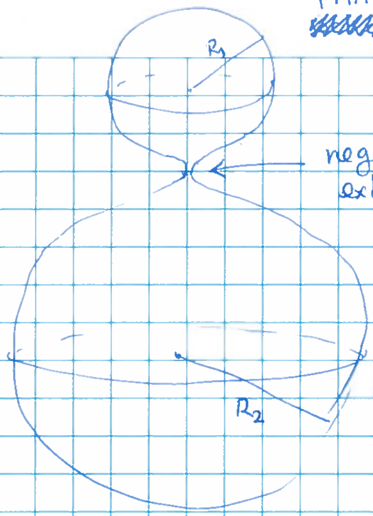
TÉCNICO LISBOA

Notes for lecture 2 by André Neves

Example:

two spheres connected by a thin neck



principal curvatures $\frac{1}{R_1}$

negative curvature extraordinarily large with respect to $1/R_1$, $1/R_2$

principal curvatures $\frac{1}{R_2}$

Slide 3

• The principal curvatures are continuous but may not be smooth. This comes from the fact that eigenvalues of a matrix depend continuously but not necessarily smoothly on the coefficients of a matrix. However, if the eigenvalues are distinct then dependence is smooth.

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This theorem means we are missing many surfaces when we consider surfaces in space. For instance surfaces of negative curvature or the flat torus can not be (isometrically) embedded in $\mathbb{R}^3$

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Question: Why did we define Gaussian and mean curvature in this way?

The normal map seems to be an important ingredient to understand the shape of a surface. Its derivative is a symmetric matrix and these two quantities - determinant and trace - are the basic invariants of such a matrix so it makes sense to consider them.

However in the end it is the theorems we can prove and in particular Gauss's Egregium Theorem that we will soon see that justifies these definitions and shows that considering these quantities is indeed a good idea.