PROBLEMS

0: Consider two smooth functions $h_i : \mathbb{R}^2 \to \mathbb{R}$ where $h_2 \geq h_1$, both with $h_1(0) = h_2(0) = 0$ and both having a critical point at the origin. Moreover, we assume that h_1 has a local minimum at the origin.

- Find an example where $\det \operatorname{Hess} h_2(0) < \det \operatorname{Hess} h_1(0)$
- Show that if $h_1 \geq 0$ then $\det \operatorname{Hess} h_2(0) \geq \det \operatorname{Hess} h_1(0)$.

1: Let S be a compact surface contained inside a unit ball and intersecting the unit sphere S^2 a point p. Show that and $K^S(p) \geq 1$.

2: Show that if a diffeomorphism $\phi : S_1 \to S_2$ is such that

$$d\phi_p(v).d\phi_p(w) = v.w \quad \text{ for all } v, w \in T_pS_1,\, p \in S_1$$

then ϕ is an isometry.

3: Show that a compact surface $S \subset \mathbb{R}^3$ with $K \geq 0$ has genus 0.