An introduction to Lie groups, symmetries, and symplectic geometry
Problem Set 3

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Exercise 1. *Let* $L : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ *be the* Riemannian metric

$$L(x, v) = g_{ij}(x)v^i v^j.$$

Show that the Euler-Lagrange equations are equivalent to the system of differential equations

$$\Big\{ \ddot{y} = -\Gamma^i_{kj}\dot{y}^k\dot{y}^j, \quad i, j, k = 1, \cdots, n,$$

where the Christoffel symbols

$$\Gamma^i_{jk} = \frac{1}{2}g^{il}\left(\frac{\partial g_{kl}}{\partial x^j} + \frac{\partial g_{il}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^l}\right), \qquad \forall i, j, k \in \{1, \cdots, n\}.$$

Exercise 2. *If* $L = \frac{1}{2}|v|^2 - V(x)$ *where* $V : \mathbb{R}^n \to \mathbb{R}$*, then the Euler-Lagrange equations become the* equation of motion in a conservative force field

$$\ddot{y} = -\text{grad } V(y).$$