Hyperbolic Geometry - from graphs to geometry
Problem Set 1

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Exercise 1. *Show that a continous map taking round circles to round circles is differentiable.*

Exercise 2. *Show that a differentiable orientation-preserving map taking round circles to round circles is holomorphic.*

Exercise 3. *Consider the statement*

"The inversion map $z \mapsto \frac{1}{\bar{z}}$ maps round circles to round circles."

Modify it to a correct version and prove that version.

Exercise 4. *Show that the* projective special unitary group

$$PSU(1,1) = \left\{ \begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix} : \alpha, \beta \in \mathbb{C}, |\alpha|^2 - |\beta|^2 = 1 \right\} / \{\pm I\}$$

is the subgroup of transformations in $PSL(2,\mathbb{C})$ which map $S^1 = \{z \in \mathbb{C} : |z| = 1\}$ to itself and preserve orientation.

Exercise 5. *Show that the* real projective special linear group

$$PSL(2,\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in GL(2,\mathbb{R}) : ad - bc = 1 \right\} / \{\pm I\}$$

is the subgroup of transformation in $PSL(2,\mathbb{C})$ which map $\mathbb{H} = \{z \in \mathbb{C} : \mathrm{Im}\, z > 0\}$ to itself.

Exercise 6. *For a hyperbolic triangle, show that*

$$\cos\alpha = \sin\beta \sin\gamma \cosh(a) - \cos\beta\cos\gamma$$

holds, where α, β, γ are the internal angles and a is the hyperbolic length of the side opposite to α.