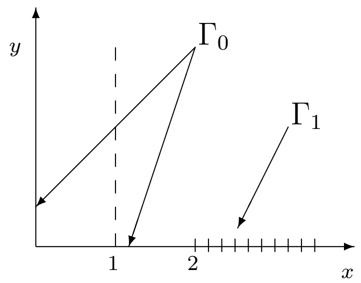
Problem 1. In the random walk game in the quarter plane of the figure, and with Γ_1 and Γ_0 being the open and closed parts of the boundary respectively, find the value of y to maximize the probability of exiting starting the walks from $(1, y)$.



Problem 2 (to be done after the fifth class).

Consider the problem

$$\begin{cases} u_{tt} - u_{xx} = 0 & \text{in} \quad (0,\pi)\times\mathbb{R}, \\ u_x(0,t) = u_x(\pi,t) = 0 & \text{for} \quad t\in\mathbb{R}, \\ u(x,0) = g(x) & \text{for} \quad x\in(0,\pi), \\ u_t(x,0) = 0 & \text{for} \quad x\in(0,\pi). \end{cases}$$

i) Make an even reflection of the problem with respect to $x = 0$, and then a periodic extension, to transform the problem into one in all of $\mathbb{R}$. Then use d'Alembert's formula to find $u(\pi/2, 5\pi)$ in terms of g.

ii) If

$$g(x) = \begin{cases} 0 & \text{if} \quad x\in[0,\pi/2] \\ 1 & \text{if} \quad x\in(\pi/2,\pi], \end{cases}$$

compute explicitely, using Fourier series, the solution of the problem.

iii) Compute $\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$ and $\sum_{k=0}^{\infty} \frac{(-1)^k \cos(2k+1)}{2k+1}$.

Problem 3 (to be done after the fifth class). Let $Q = \{(x,y)\in\mathbb{R}^2 : a < x < b, 0 < y < c\}$, $D_1 = \partial Q \cap \{x = a\}$, $D_2 = \partial Q \cap \{x = b\}$, $D_3 = \partial Q \cap \{y = 0\}$, and $D_4 = \partial Q \cap \{y = c\}$. Consider the problem

$$\begin{cases} \Delta u = 0 & \text{in } Q, \\ u = 0 & \text{on } D_1, D_3 \text{ i } D_4, \\ u = d(y) & \text{on } D_2, \end{cases}$$

where $d(y)$ is a given function in $[0, c]$.

(a) Give necessary conditions on d for the solution to be of class C^2 in $\overline{Q}$.

(b) If $d(y) = \sin(2\pi y/c)$, determine the sign of $u_y((a+b)/2, c/2)$ (without making any explicit computation).

[Next two questions can be done independently of having answered or not the two previous ones]

(c) For a general $d = d(y)$, find the solution using separation of variables and Fourier series.

(d) How would you solve the same problem with Dirichlet conditions on all ∂Q, that is: $u = d_1(y)$ on D_1, $u = d_2(y)$ on D_2, $u = d_3(x)$ on D_3 and $u = d_4(x)$ on D_4?