Mathematical Modeling in Hydrodynamics; A. Nachbin

Problem Set 2: consider ALL PROBLEM in 2D

(1): Check that the Joukowsky conformal mapping

$$w = f(z) = z + \frac{\lambda^2}{z}$$

maps a circle onto an ellipse. What is the choice of λ for it to map a circle to a double-sided slit (cut).

Put all this together for the flow (as presented in class with the Circle Theorem) regarding an elliptical cylinder.

(2): On a strip of unit width take the equation

$$\beta\phi_{xx} + \phi_{yy} = 0,$$

($\beta \ll 1$) together with the Neumann condition $d\phi/dy = 0$ at $y = 0$.

(A): Solve this problem (formally; because we do not prove that the series converges) using

$$\phi(x, y) = \sum_{n=0}^{\infty} y^n f_n(x, t).$$

Recall that the recurrence relation only couples odd terms in f with odds terms, and even terms with even terms.

(B): Now without using the power series expansion, solve the problem above with the Dirichlet data $\phi(x, 1) = \exp(ix)$. Try an ansatz in a similar fashion as I did in class 2, in the spirit of separation of variables.

(C) Now add the linear free surface condition at $y = 1$

$$\phi_{tt} + \frac{1}{\beta}\phi_y = 0.$$

Do you know where this came from? In any case, solve this time dependent problem in $\phi(x, y, t)$ along the lines of above.