Inversão de uma matriz pelo método de Gauss-Jordan:

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 2 \\ 4 & 3 & 5 \end{bmatrix}$$

Implementação:

$$\color{blue}{[\, A \vdots I \,]} \quad = \quad \left[\begin{array}{ccc|ccc} 2 & 1 & 3 & 1 & 0 & 0 \\ 1 & -1 & 2 & 0 & 1 & 0 \\ 4 & 3 & 5 & 0 & 0 & 1 \end{array}\right].$$

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}; \qquad \xrightarrow{E_1} \left[\begin{array}{ccc|ccc} 2 & 1 & 3 & 1 & 0 & 0 \\ 0 & -3/2 & 1/2 & -1/2 & 1 & 0 \\ 0 & 1 & -1 & -2 & 0 & 1 \end{array}\right]$$

$$E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2/3 & 1 \end{bmatrix}; \qquad \xrightarrow{E_2} \left[\begin{array}{ccc|ccc} 2 & 1 & 3 & 1 & 0 & 0 \\ 0 & -3/2 & 1/2 & -1/2 & 1 & 0 \\ 0 & 0 & -2/3 & -7/3 & 2/3 & 1 \end{array}\right]$$

$$F_3 = \begin{bmatrix} 1 & 0 & 9/2 \\ 0 & 1 & 3/4 \\ 0 & 0 & 1 \end{bmatrix}; \qquad \xrightarrow{F_3} \left[\begin{array}{ccc|ccc} 2 & 1 & 0 & -19/2 & 3 & 9/2 \\ 0 & -3/2 & 0 & -9/4 & 3/2 & 3/4 \\ 0 & 0 & -2/3 & -7/3 & 2/3 & 1 \end{array}\right]$$

$$F_2 = \begin{bmatrix} 1 & 2/3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \qquad \xrightarrow{F_2} \left[\begin{array}{ccc|ccc} 2 & 0 & 0 & -11 & 4 & 5 \\ 0 & -3/2 & 0 & -9/4 & 3/2 & 3/4 \\ 0 & 0 & -2/3 & -7/3 & 2/3 & 1 \end{array}\right]$$

$$D^{-1} = \begin{bmatrix} 1/2 & 0 & 0 \\ 0 & -2/3 & 0 \\ 0 & 0 & -3/2 \end{bmatrix}; \qquad \xrightarrow{D^{-1}} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -11/2 & 2 & 5/2 \\ 0 & 1 & 0 & 3/2 & -1 & -1/2 \\ 0 & 0 & 1 & 7/2 & -1 & -3/2 \end{array}\right] = \color{blue}{[\, I \vdots A^{-1} \,]}$$

Conclusão: A matriz inversa de A, A^{-1}, é expressa como um produto de matrizes elementares,

$$A^{-1} = D^{-1} F_2 F_3 E_2 E_1 \; .$$